Royal Higher Secondary School - Dhoraji

Date : 18-02-25
Marks : 50
Time : 1:00 Hour

Std - 12 **Sub. : Maths Full Course**

Part :- A [50]

(1) If $f : R-\{-1\} \rightarrow R-\{-1\}, f(x)=\frac{1-x}{1+x}$ then $f^{-1}(x)=$.......

(A) $\frac{x+1}{x-1}$ (B) $\frac{1-x}{1+x}$ (C) $\frac{x-1}{x+2}$ (D) $\frac{1+x}{1-x}$

(2) A relation R on set A={1,2,3,4} is given as R={(1,3), (4,2), (2,4), (2,3), (3,1)} then R is ......

(A) reflexive (B) transitive (C) not symmetric (D) a function

(3) Let E={1,2,3,4} and F={1,2}are given then number of onto function from E to F is..........

(A) 14 (B) 16 (C) 12 (D) 8

(4) $\sin^{-1}\left(\sin\frac{7\pi}{6}\right)=$.......

(A) $\frac{\pi}{6}$ (B) $\frac{5\pi}{6}$ (C) $-\frac{\pi}{6}$ (D) $\frac{7\pi}{6}$

(5) Value of $\cos\left[\frac{\pi}{6}+\cos^{-1}\left(-\frac{1}{2}\right)\right]$ is ....

(A) $-\frac{\sqrt{3}}{2}$ (B) $\frac{\sqrt{3}-1}{2\sqrt{2}}$ (C) $\frac{\sqrt{5}-1}{4}$ (D) $\frac{\sqrt{3}+1}{2\sqrt{2}}$

(6) If $\cot^{-1}x-\cot^{-1}y=0$ and $\cos^{-1}x+\cos^{-1}y=\frac{\pi}{2}$ then x+y=........ .

(A) 0 (B) 1 (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

(7) If $A=\begin{bmatrix}3 & 4\\5 & 7\end{bmatrix}$, then A(adj A)=........ .

(A) I (B) $|A|$ (C) $|A|I$ (D) none of these

(8) If A and B are congruent square matrix then $(A-B)^2=$........ .

(A) $A^2-AB-BA+B^2$ (B) $A^2-2AB+B^2$

(C) A^2-B^2 (D) A^2+B^2

(9) If $A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 3\alpha \\ 0 & 0 & 5 \end{bmatrix}$ and $|A^2| = 25$ then $|\alpha| = ...$

(A) 1 (B) $\frac{1}{5}$ (C) 5 (D) 5^2

(10) Area of triangle having vertices (1,1), (3,3) and (5,k) is 2 unit then k =........

(A) 3 or 7 (B) 2 or 3 (C) 4 or 7 (D) 3 or 4

(11) If $D = \begin{vmatrix} 2 & \cos\theta & 2 \\ -\cos\theta & 2 & \cos\theta \\ -2 & -\cos\theta & 2 \end{vmatrix}$, then D lies in .... interval.

(A) $(16,\infty)$ (B) (16,20) (C) [12,16] (D) [16,20]

(12) Value of $\begin{vmatrix} 441 & 442 & 443 \\ 445 & 446 & 447 \\ 449 & 450 & 451 \end{vmatrix} = ...$

(A) $441 \times 446 \times 451$ (B) 0

(C) −1 (D) 1

(13) If $f(x) = \begin{cases} x & ,0 \le x \le 1 \\ 2x-1 & ,1 < x \end{cases}$, then ....

(A) f is discountious at x=1

(B) f is diffrentatible at x=1

(C) f is countious at x=1 & not diffrentatible

(D) none of these

(14) $\frac{d}{dx}\tan^{-1}\frac{1-x}{1+x} = ...$

(A) $\frac{-1}{1+x^2}$ (B) $\frac{1}{1+x^2}$ (C) $\frac{1+x}{1-x}$ (D) $\frac{2}{1+x^2}$

(15) If x=2bt, y=bt², then $\frac{d^2y}{dx^2} = ...$

(A) 1 (B) $\frac{1}{2b}$ (C) $-\frac{1}{2bt^3}$ (D) $\frac{1}{2bt^3}$

(16) If $y = \log\left(\sec\left(e^{x^2}\right)\right)$, then $\frac{dy}{dx} = ...$

(A) $x^2 e^{x^2} \tan\left(e^{x^2}\right)$ (B) $e^{x^2}.\tan e^{x^2}$ (C) $2x\left(\tan e^{x^2}\right)e^{x^2}$ (D) $2xe^{x^2}.\sec e^{x^2} \tan e^{x^2}$

(17) Two sides of triangle are 4 and 5 m respectively and the angle between then is increasing at rate of 0.05 rad/sec. When the angle between sides is $\frac{\pi}{3}$ then find the rate of increasing in it's area ......

(A) 0.25 (B) 0.5 (C) $0.25\sqrt{3}$ (D) $0.5\sqrt{3}$

(18) $f(x) = |x-2| + 3|x-4|$ is ...... .in (2,4) , $x \in R$

(A) decreasing (B) increasing (C) constant (D) can't determine

(19) The local minimum value of $\frac{x}{\log x}$ is .... $x \in R^+$

(A) -1 (B) 0 (C) $\frac{1}{e}$ (D) e

(20) If ....then $f(x) = |x-1| + |x-2|$ is an increasing function.

(A) x>2 (B) x<1 (C) x<0 (D) x<−2

(21) $\int \frac{(5+\log x)}{(6+\log x)^2} dx = ... + c.$

(A) $\frac{x}{\log_e x + 6}$ (B) $\frac{1}{5 + \log_e x}$ (C) $\frac{x}{\log_e x + 5}$ (D) $\frac{e^x}{\log_e x + 6}$

(22) $\int \left(\frac{x-2}{x^3}\right) e^x \, dx = ... + c.$

(A) $-\frac{2e^x}{x^3}$ (B) $\frac{e^x}{x^2}$ (C) $-\frac{e^x}{x^2}$ (D) $\frac{2e^x}{x^3}$

(23) $\int \sqrt{x^2 - 4x + 2} \, dx = ... + c.$

(A) $\frac{x-2}{2}\sqrt{x^2 - 4x + 2} + \log\left|x - 2 + \sqrt{x^2 - 4x + 2}\right|$

(B) $\frac{x-2}{2}\sqrt{x^2 - 4x + 2} - \log\left|x - 2 + \sqrt{x^2 - 4x + 2}\right|$

(C) $\frac{x-2}{2}\sqrt{x^2 - 4x + 2} + \sin^{-1}\left(\frac{x-2}{2}\right)$

(D) $\frac{x-2}{2}\sqrt{x^2 - 4x + 2} + \frac{1}{2}\sin^{-1}\left(\frac{x-2}{2}\right)$

(24) If $\int_0^a \frac{1}{1+4x^2}\,dx = \frac{\pi}{8}$ then a=...........

(A) $\frac{\pi}{2}$ (B) $\frac{\pi}{4}$ (C) $\frac{1}{2}$ (D) 1

(25) $\int_{-1}^{1} \sin^3 x \cos^4 x \, dx = ...$

(A) 0 (B) 1 (C) π (D) 2π

(26) $\int_0^{\frac{\pi}{2}} \frac{\cos 2x}{(\sin x + \cos x)^2}\,dx = ...$

(A) $\frac{-\pi}{4}$ (B) $\frac{\pi}{4}$ (C) $\frac{\pi}{2}$ (D) 0

(27) $\int \frac{dx}{x\sqrt{3+\log x}} = ... + c.$

(A) $2\sqrt{3+\log x}$ (B) $\frac{2}{\sqrt{3+\log x}}$ (C) $\sqrt{3+\log x}$ (D) $-2\sqrt{3+\log x}$

(28) $\int e^{3\log x}.\left(x^4+1\right)^{-1}\,dx = ... + c.$

(A) $\log(x^4+1)$ (B) $-\log(x^4+1)$ (C) $\frac{1}{4}\log\left(x^4+1\right)$ (D) $\frac{-3}{\left(x^4+1\right)^2}$

(29) $\int \frac{10x^9 + 10^x \log 10}{10^x + x^{10}}\,dx = ... + c.$

(A) $10^x - x^{10}$ (B) $10^x + x^{10}$ (C) $(10^x - x^{10})^{-1}$ (D) $\log|10^x + x^{10}|$

(30) $\int \frac{1+x+\sqrt{x+x^2}}{\sqrt{x}+\sqrt{1+x}}\,dx = ...$

(A) $\frac{1}{2}\sqrt{1+x}+c$ (B) $\frac{2}{3}(1+x)^{\frac{3}{2}}+c$ (C) $\sqrt{1+x}+c$ (D) $2(x+1)^{\frac{3}{2}}+c$

(31) Area under the curve $y=2x-x^2$ and X–axis.

(A) $\frac{8}{5}$ (B) 2 (C) 8 (D) $\frac{4}{3}$

(32) Area between the Curve y=tan x, X–axis and lines $x=0$ & $x=\frac{\pi}{4}$ is ......

(A) log 2 (B) $\frac{3}{2}\log 2$ (C) $\frac{1}{2}\log 2$ (D) 2 log 2

(33) Area between the Curve $y=|x-5|$, X–axis and lines x=5, x=6 is ....

(A) 0.50 (B) 0.25 (C) 1.25 (D) 0.75

(34) Degree of homogeneous function $f(x,y)=\frac{x^3-y^3}{x+y}$ is ....

(A) 1 (B) 2 (C) 3 (D) undefined

(35) Integrating factor of differential equation $\frac{dy}{dx}=\frac{1}{x+y+2}$ is ....

(A) e^x (B) e^{x+y+2} (C) e^{-y} (D) $\log|x+y+2|$

(36) Number of arbitary constant in a second order differential equation will be......

(A) 4 (B) 2 (C) 1 (D) 0

(37) Order and Degree of differential equation $\frac{d^2y}{dx^2}=\left[1+\left(\frac{dy}{dx}\right)^2\right]^{\frac{3}{2}}$ is respectively.

(A) 6, 1 (B) 3, 2 (C) 2, 2 (D) 1, 1

(38) $|\vec{x}|=|\vec{y}|=1, \vec{x}\perp\vec{y}$ then $|\vec{x}+\vec{y}|=$........

(A) $\sqrt{3}$ (B) $\sqrt{2}$ (C) 1 (D) 0

(39) If $\vec{x}=(a,4,2a)$ and $\vec{y}=(2a,-1,a)$ are mutually perpendicular then a=....... .

(A) 2 (B) 1 (C) 4 (D) કોઈ પણ વાસ્તવિક સંખ્યા

(40) If $|\vec{x}.\vec{y}|=\cos\alpha$ then $|\vec{x}\times\vec{y}|=$........

(A) $\pm\sin\alpha$ (B) $\sin\alpha$ (C) $-\sin\alpha$ (D) $\sin^2\alpha$

(41) Unit vector along the direction of sum of vectors (1,1,1), (2,–1,–1) and (0,2,6) is ....

(A) $-\frac{1}{7}(3,2,6)$ (B) $\frac{1}{49}(3,2,6)$ (C) $\frac{1}{7}(3,-2,6)$ (D) $\frac{1}{7}(3,2,6)$

(42) Equation of line passing through origin and having directional angle $\frac{2\pi}{3},\frac{\pi}{4},\frac{\pi}{3}$..

(A) $x=\frac{y}{-\sqrt{2}}=z$ (B) $\frac{x}{-1}=\frac{y}{-\sqrt{2}}=z$

(C) $x=\frac{y}{-\sqrt{2}}=-z$ (D) $x=\frac{y}{\sqrt{2}}=z$

(43) Angle between lines $x = k+1, y = 2k-1, z = 2k+3;\ k \in R$ and $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-1}{-2}$ will be.........

(A) $\sin^{-1}\frac{4}{3}$ (B) $\cos^{-1}\frac{4}{9}$ (C) $\sin^{-1}\frac{\sqrt{5}}{3}$ (D) $\frac{\pi}{2}$

(44) If lines $\vec{r} = (2,0,3) + k(7,-a,6), k \in R$ and $\vec{r} = (5,1,2) + k(a,a,3), k \in R$ are mutually perpendicular then a =.......... .

(A) 6 (B) −9 (C) 2 (D) −2

(45) Equation of lines passing through (2,3,4) and $\frac{x-1}{3} = \frac{2-y}{-5} = \frac{z-10}{15}$ parallel to line is ....

(A) $\vec{r} = (2+3k, 3+5k, 4+15k), k \in R$ (B) $\vec{r} = (2-3k, 3-5k, 4-15k), k \in R$

(C) $\vec{r} = (2+3k, 3-5k, 4+15k), k \in R$ (D) none of these

(46) Let A and B are two events where $P(A) = 0.4$, $P(A \cup B) = 0.7$ and $P(B) = p$. If A and B are independent event then p = ...

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{3}{4}$ (D) $\frac{5}{6}$

(47) Three coins are tossed together. If first coin shows head then probability that remaining two coins will also shows head will be ....

(A) $\frac{1}{4}$ (B) $\frac{1}{2}$ (C) $\frac{1}{8}$ (D) 1

(48) If $P(A) = \frac{2}{3}, P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{5}{6}$ then A and B are.......

(A) mutually exclusive (B) mutually exclusive & independent

(C) independent (D) dependent

(49) For the bounded fesiable region whose corner points are(0,10), (5,5), (15,15), (0,20) and let z=px+qy, where p, q>0 having maximum value of z at (15,15) and (0,20) then relationship between p and q ....

(A) p=q (B) p=2q (C) q=2p (D) q=3p

(50) (49) In LPP problem, objective function will be ....

(A) constant (B) function whose maximum & minimum values need to find

(C) inequality (D) quadratic equation

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*** Best of Luck ***

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Part :- B [50]

SECTION :- A [16]

❍ **Answer the following questions [Any eight] : [2 marks each]**

(1) Write in simplest from $\tan^{-1}\left(\frac{\cos x}{1-\sin x}\right), -\frac{\pi}{2} < x < \frac{\pi}{2}$

(2) Prove that $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

(3) If $f(x) = \begin{cases} \frac{k\cos x}{\pi - 2x} & , x \neq \frac{\pi}{2} \\ 3 & , x = \frac{\pi}{2} \end{cases}$, is countious at $x = \frac{\pi}{2}$ then find value of k.

(4) Find the integration of $\frac{\sin^2 x}{1+\cos x}$ with respect to x.

(5) Find the area bounded by the line $y=3x+2$, X–axis and line $x=-1$ & $x=1$.

(6) Find the area of the circle $x^2+y^2=a^2$ using integration.

(7) If two vectors $\vec{a}$ and $\vec{b}$ having same magnitude, angle between them in 60^{o} and their dot product is $\frac{1}{2}$ then find the magnitude of both vectors.

(8) For $\vec{a}, \vec{b}$ & $\vec{c}$, $|\vec{a}| = 3, |\vec{b}| = 4, |\vec{c}| = 5$ and each vector is perpendicular to sum of other two vectors then find $|\vec{a} + \vec{b} + \vec{c}|$

(9) Find the angle between lines $\vec{r} = 3\hat{i} + 2\hat{j} - 4\hat{k} + \lambda(\hat{i} + 2\hat{j} + 2\hat{k})$ and $\vec{r} = 5\hat{i} - 2\hat{j} + \mu(3\hat{i} + 2\hat{j} + 6\hat{k})$.

(10) Product of slope of tangent at (x, y) of any curve and it's y-cordinate is equal to it's x-cordinate. If the curve is passing through (0, -2) then find the equation of curve.

(11) A dice is rolled twice and sum of number appeared is equal to 6 find the probability that at least 4 appear once

(12) Probability of A and B solve a problem independently is $\frac{1}{2}$ and $\frac{1}{3}$. If A and B solve problem independently then find probability of :

(i) problem is solved (ii) any one of A or B solve the problem

SECTION :- B [18]

- **Answer the following questions [Any six] : [3 marks each]**

(13) Let X={1,2,3,4,5,6,7,8,9}. and R_1 is a relation on X given by
$R_1 = \{(x,y): x-y \text{ is divisible by } 3\}$ and another relation R_2 on X given by
$R_2 = \{(x,y):\{x,y\}\subset\{1,4,7\} \text{ or } \{x,y\}\subset\{2,5,8\} \text{ or } \{x,y\}\subset\{3,6,9\}\}$
Prove that $R_1=R_2$.

(14) Find the differentation of $x^{\sin x}+(\sin x)^{\cos x}$ with respect to x.

(15) Evaluate : $\int \frac{(x^2+x+1)\,dx}{(x+2)(x^2+1)}$

(16) If $A=\begin{bmatrix}3 & -2\\ 4 & -2\end{bmatrix}$ and $I=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ are given then find the value of k for which $A^2=kA-2I$

(17) $\begin{aligned}2x+3y+3z&=5\\ x-2y+z&=-4\\ 3x-y-2z&=3\end{aligned}$ solve the given equation using matrix method

(18) Find the global maximum and global minimum value of the function $f(x)=\cos^2 x+\sin x, x\in[0,\pi]$

(19) Find the shortest distance between lines $\vec{r}=(\hat{i}+2\hat{j}+\hat{k})+\lambda(\hat{i}-\hat{j}+\hat{k})$ and $\vec{r}=2\hat{i}-\hat{j}+\hat{k}+\mu(2\hat{i}+\hat{j}+2\hat{k})$

(20) If the direction of a unit vector is in the directoin of sum of vectors $2\hat{i}+4\hat{j}-5\hat{k}$ and $\lambda\hat{i}+2\hat{j}+3\hat{k}$ and the dot product of this vector with $\hat{i}+\hat{j}+\hat{k}$ is equal to1 then find λ.

(21) Find the minimum and maximum value of $Z=x+2y$ under the condition $x+2y\geq 100, 2x-y\leq 0, 2x+y\leq 200, x,y\geq 0$

SECTION :- C [16]

❍ **Answer the following questions [Any four] : [4 marks each]**

(22) If $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$, then show that $A \text{ adj } A = |A| I$ also find A^{-1} using this result.

(23) If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is a identity matrix of 2nd order then prove that

$$I + A = (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}.$$

(24) Evaluate : $\int\left(\sqrt{\cot x} + \sqrt{\tan x}\right) dx$

(25) Find the value of $\int_0^{\pi} \frac{x\, dx}{a^2\cos^2 x + b^2\sin^2 x}$

(26) Find the solution of differential equation :

$$(x\, dy - y\, dx)\, y \sin\left(\frac{y}{x}\right) = (y\, dx + x\, dy)\, x \cos\left(\frac{y}{x}\right)$$

(27) A rectangular sheet of tin 3 m × 8 m is to be made into a box without top, by cutting off square from each corner and folding up the flap. What should be the side of the square to be cut of show that volume of the box of maximum.

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* **Best of Luck** *

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